

1.- Resolver las siguientes ecuaciones exponenciales:

(2 puntos)

a) $9^{2x-1} + 5 = 8 \cdot 3^{2x-1}$

b) $4^{3x} = 5^{x-1}$

c) $e^{\ln 2x} = -\log_{\frac{1}{5}} 25$

(a) $9 \cdot 9^{-1} + 5 = 8 \cdot 3^{2x} \cdot 3^{-1} \Rightarrow \frac{3^{4x}}{9} + 5 - \frac{8}{3} \cdot 3^{2x} = 0$ $t = 3^{2x}$

$\frac{1}{9}t^2 - \frac{8}{3}t + 5 = 0 \Rightarrow t^2 - 24t + 45 = 0$

$t = \frac{24 \pm 20}{2}$
 $t = 22 \Rightarrow 3^{2x} = 22 \Rightarrow 2x \cdot \log 3 = \log 22 \Rightarrow x = \frac{\log 22}{2 \log 3} = \frac{\log 22}{\log 9}$ //
 $t = 2 \Rightarrow 3^{2x} = 2 \Rightarrow 2x \cdot \log 3 = \log 2 \Rightarrow x = \frac{\log 2}{2 \log 3} = \frac{\log 2}{\log 9}$ //

(b) $\log 4^{3x} = \log 5^{x-1} \Rightarrow 3x \log 4 = (x-1) \log 5 \Rightarrow 3x \log 4 = x \log 5 - \log 5 \Rightarrow$

$3x \log 4 - x \log 5 = -\log 5 \Rightarrow x(3 \log 4 - \log 5) = -\log 5 \Rightarrow x = \frac{-\log 5}{3 \log 4 - \log 5}$ //

(c) $\log_{\frac{1}{5}} 25 = x \Rightarrow \left(\frac{1}{5}\right)^x = 25 \Rightarrow 5^{-x} = 5^2 \Rightarrow -x = 2 \Rightarrow x = -2.$

$e^{\ln 2x} = -\log_{\frac{1}{5}} 25 \Rightarrow e^{\ln 2x} = +2 \Rightarrow \ln e^{\ln 2x} = \ln 2 \Rightarrow \ln 2x \cdot \overset{1}{e} = \ln 2$

$\Rightarrow \ln 2x = \ln 2 \Rightarrow 2x = 2 \Rightarrow x = 1$ //

2.- Resolver la ecuación logarítmica:

$(x^2 - 4x + 7) \log 5 + \log 16 = 4$

(1 punto)

$\log 5^{x^2-4x+7} + \log 16 = \log 10^4$

$\log 5^{x^2-4x+7} \cdot 16 = \log 10^4$

$5^{x^2-4x+7} \cdot 16 = 10^4$

$5^{x^2-4x+7} = \frac{2 \cdot 5^4}{2^4}$

$x^2 - 4x + 7 = 4 \Rightarrow x^2 - 4x + 3 = 0$

$x = \frac{4 \pm \sqrt{16-12}}{2}$

$x_1 = 3$ // válida

$x_2 = 1$ // válida

3.-Hallar el **dominio** de definición de las siguientes funciones:

(2,5 puntos)

a) $f(x) = \sqrt{\frac{x^4 + x^2 - 2}{2-x}}$

b) $f(x) = \frac{2x-1}{\sqrt{x^5 + 2x^3}}$

c) $f(x) = \frac{2\ln(x^2-1)+x}{x-3}$

d) $f(x) = \frac{\sqrt{x^2-2x}}{\sqrt{x+4}}$

(a) $\frac{x^4 + x^2 - 2}{2-x} \geq 0 \Rightarrow \text{Sig} \frac{(x-1)(x+1)(x^2+2)}{(2-x)}$

$x^4 + x^2 - 2 = 0 \Rightarrow t = x^2 \Rightarrow t^2 + t - 2 = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+8}}{2}$
 $t = 1 \Rightarrow x^2 = 1 \Rightarrow \begin{cases} x = 1 \\ x = -1 \end{cases}$
 $t = -2 \Rightarrow x^2 = -2 \Rightarrow \text{no sol}$

$\text{Dom } f(x) = (-\infty, -1] \cup [1, 2)$

(b) $x^5 + 2x^3 > 0 \Rightarrow x^3(x^2 + 2) > 0$ $\text{Sig } x^3(x^2 + 2)$

$\text{Dom } f(x) = [0, +\infty)$ ($x^2 + 2 = 0 \Rightarrow x^2 = -2 \Rightarrow \text{no sol}$)

(c) $f(x) = \frac{2\ln(x^2-1)+x}{x-3}$

$x^2 - 1 > 0 \Rightarrow \text{Sig } (x-1)(x+1)$

$D\ln = (-\infty, -1) \cup (1, \infty)$

$x - 3 = 0 \Rightarrow x = 3$ quitamos

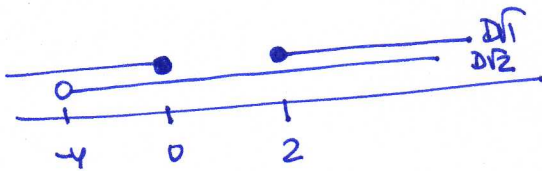
$\text{Dom } f(x) = (-\infty, -1) \cup (1, 3) \cup (3, +\infty)$

(d) $f(x) = \frac{\sqrt{x^2 - 2x}}{\sqrt{x+4}}$

$x(x-2) \geq 0$ $\text{Sig } x(x-2)$

$D\sqrt{1} = (-\infty, 0] \cup [2, \infty)$

$D\sqrt{2} = [-4, \infty)$



$\text{Dom } f(x) = (-4, 0] \cup [2, +\infty)$

