



Apellidos CORRECCIÓN

Nombre \_\_\_\_\_

Grupo \_\_\_\_\_

CALIFICACIÓN

1.- Resolver la ecuación trigonométrica:

$$2 \cos x + 4 \operatorname{sen} \frac{x}{2} = 3$$

2.- Sabiendo que  $\cos 2x = -\frac{2}{3}$ ,  $\frac{\pi}{2} < 2x < \pi$ . Calcula, utilizando fórmulas trigonométricas y sin calculadora:

- a)  $\cos(180^\circ - x)$       b)  $\operatorname{tag} 4x$

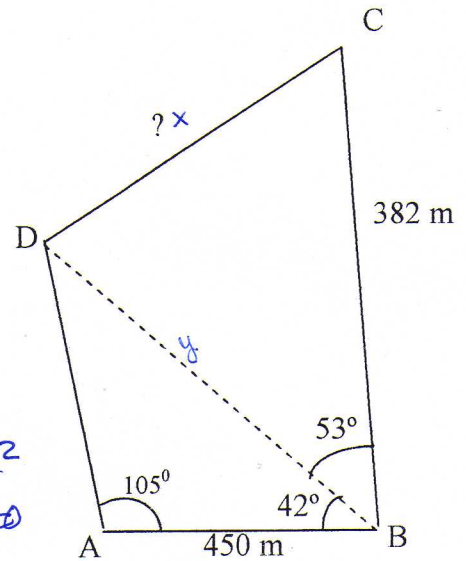
3.- Demostrar que se verifica la siguiente identidad:

$$\frac{1}{1 + \operatorname{tag} x \cdot \operatorname{tag} 2x} = \cos 2x$$

4.- Resuelve, **sin calculadora**, el triángulo ABC y calcula su área, sabiendo que

$A = 60^\circ$ ,  $B = 75^\circ$  y  $c = \sqrt{2}$

5.- Halla la distancia que hay entre dos barcos C y D. Sabiendo que la distancia que entre A y B es de 450 m y la distancia entre B y C es de 382 m. Además, con el teodolito hemos obtenido los valores de los ángulos:  $BAD = 105^\circ$ ,  $ABC = 42^\circ$  y  $CBD = 53^\circ$



CORRECCIÓN

1.-

$$2 \cos x + 4 \operatorname{sen} \frac{x}{2} = 3 \Rightarrow$$

$$2 \cos x + 4 \left( \pm \sqrt{\frac{1 - \cos x}{2}} \right) = 3 \Rightarrow 4 \left( \pm \sqrt{\frac{1 - \cos x}{2}} \right) = 3 - 2 \cos x \Rightarrow$$

$$16 \cdot \left( \frac{1 - \cos x}{2} \right) = (3 - 2 \cos x)^2 \Rightarrow 8(1 - \cos x) = 9 + 4 \cos^2 x - 12 \cos x \Rightarrow$$

$$4 \cos^2 x - 4 \cos x + 1 = 0 \Rightarrow \cos x = \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{1}{2}$$

$$x = \arccos \left( \frac{1}{2} \right)$$

I  $x_1 = 60^\circ + 360^\circ k // x_1 = \frac{\pi}{3} + 2k\pi //$   
 II  $x_2 = 300^\circ + 360^\circ k // x_2 = \frac{5\pi}{3} + 2k\pi //$

Comprobación:

$x = 60^\circ \Rightarrow 2 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2} = 3 \Rightarrow 3 = 3 \checkmark$  válida

$x = 300^\circ \Rightarrow 2 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2} = 3 \Rightarrow 3 = 3 \checkmark$  válida.

2.  $\cos 2x = -\frac{2}{3}$   
 $\frac{\pi}{2} < 2x < \pi$

(a)  $\cos(180-x) = \ominus = -\cos x = -\frac{\sqrt{6}}{6}$  (\*)

(b)  $\operatorname{tg} 4x = \operatorname{tg} 2 \cdot 2x = \frac{2 \operatorname{tg} 2x}{1 - \operatorname{tg}^2 2x} = \frac{2 \cdot \frac{-\sqrt{5}}{2}}{1 - \frac{5}{4}} = \frac{\sqrt{5}}{-\frac{1}{4}} = -4\sqrt{5}$

Previs:

(\*)  $\cos 2x = -\frac{2}{3}$

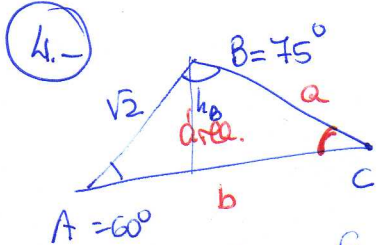
$\cos^2 x - \sin^2 x = -\frac{2}{3} \Rightarrow 2\cos^2 x - 1 = -\frac{2}{3} \Rightarrow 2\cos^2 x = \frac{1}{3} \Rightarrow \cos^2 x = \frac{1}{6} \Rightarrow \cos x = \frac{\sqrt{6}}{6}$ ,  $\frac{\pi}{4} < x < \frac{\pi}{2}$

(\*)  $\sin^2 2x = 1 - \cos^2 2x \Rightarrow \sin^2 2x = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow \sin 2x = \frac{\sqrt{6}}{3}$  (I)  $\Rightarrow \operatorname{tg} 2x = \frac{\frac{\sqrt{6}}{3}}{-\frac{2}{3}} = -\frac{\sqrt{6}}{2}$

3.  $\frac{1}{1 + \operatorname{tg} x \cdot \operatorname{tg} 2x} = \cos 2x \Rightarrow \frac{1}{1 + \operatorname{tg} x \cdot \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}} = \cos 2x \Rightarrow \frac{1}{\frac{1 - \operatorname{tg}^2 x + 2 \operatorname{tg}^2 x}{1 - \operatorname{tg}^2 x}} = \cos 2x \Rightarrow$

$\Rightarrow \frac{1 - \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x} = \cos 2x \Rightarrow \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \cos 2x \Rightarrow \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}} = \cos 2x \Rightarrow$

$\Rightarrow \cos^2 x - \sin^2 x = \cos 2x$  *cf. d*



$\hat{C} = 180 - (60 + 75) = 45$

Cálculo de a:  $\frac{a}{\operatorname{sen} 60} = \frac{\sqrt{2}}{\operatorname{sen} 45} \Rightarrow a = \frac{\sqrt{2} \cdot \operatorname{sen} 60}{\operatorname{sen} 45} = \frac{\frac{\sqrt{2} \cdot \sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} = \sqrt{3}$

Cálculo de b:  $\frac{\sqrt{2}}{\operatorname{sen} 45} = \frac{b}{\operatorname{sen} 75} \Rightarrow b = \frac{\sqrt{2} \cdot \operatorname{sen} 75}{\operatorname{sen} 45} = \frac{\frac{\sqrt{2} \cdot \frac{\sqrt{2}}{2} (\sqrt{3} + 1)}{2}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2} (\sqrt{3} + 1) = \frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2}$

$\bullet \operatorname{sen} 75 = \operatorname{sen}(45 + 30) = \operatorname{sen} 45 \cdot \cos 30 + \cos 45 \cdot \operatorname{sen} 30 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4} (\sqrt{3} + 1)$

Área:  $A = \frac{1}{2} \cdot h_b \cdot b = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} (\sqrt{3} + 1) \cdot \frac{\sqrt{6}}{2} = \frac{2\sqrt{3}}{8} (\sqrt{3} + 1) = \frac{\sqrt{3}}{4} (\sqrt{3} + 1)$  |  $\operatorname{sen} 60 = \frac{h_b}{\sqrt{2}} \Rightarrow h_b = \frac{\sqrt{2} \cdot \sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

$A = \frac{3}{4} + \frac{\sqrt{3}}{4}$

5. Cálculo de y: (no dibuj).  $\hat{BDA} = 33^\circ$ . Terna del seno

$\frac{450m}{\operatorname{sen} 33} = \frac{y}{\operatorname{sen} 103} \Rightarrow y = \frac{450 \cdot \operatorname{sen} 103}{\operatorname{sen} 33} \approx 798,08 \text{ cm}$

Cálculo de x: terna del coseno.

$x^2 = y^2 + 132^2 - 2 \cdot 382 \cdot y \cos 53 \Rightarrow x^2 = \sqrt{415818,22} \approx 644,8 \text{ cm}$