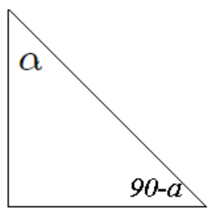
Identidades Trigonométricas

(Identidades tomadas de pruebas de cátedra de MA0125)

Angulos complementarios

Una función trgonométrica de un ángulo agudo α es igual a la cofunción del ángulo complementario de α.



$\sin\alpha = \cos(90-\alpha)$ $\qquad$ $\csc\alpha = \sec(90-\alpha)$

$\cos\alpha = \sin(90-\alpha)$ $\qquad$ $\sec\alpha = \csc(90-\alpha)$

$\tan\alpha = \cot(90-\alpha)$ $\qquad$ $\cot\alpha = \tan(90-\alpha)$

Identidades Trigonométricas Básicas	Identidades Trigonométricas Pitagóricas
$\tan x = \dfrac{\sin x}{\cos x}$	$\sin^2 x + \cos^2 x = 1$
$\cot x = \dfrac{\cos x}{\sin x}$	$\sin^2 x = 1 - \cos^2 x$
$\cot x = \dfrac{1}{\tan x}$	$\cos^2 x = 1 - \sin^2 x$
$\sec x = \dfrac{1}{\cos x}$	$\tan^2 x + 1 = \sec^2 x$
$\csc x = \dfrac{1}{\sin x}$	$\cot^2 x = \csc^2 x - 1$

Paridad Identidades Trigonométricas

$sen(-\alpha) = -\sin\alpha$ $\qquad$ $\csc(-\alpha) = -\csc\alpha$

$\cos(-\alpha) = \cos\alpha$ $\qquad$ $\sec(-\alpha) = \sec\alpha$

$\tan(-\alpha) = -\tan\alpha$ $\qquad$ $\cot(-\alpha) = -\cot\alpha$

Identidades Trigonométricas para suma y resta de ángulos

1. $\sin(a \pm b) = \sin a \cdot \cos b \pm \sin b \cdot \cos a$
2. $\cos(a \pm b) = \cos a \cdot \cos b \mp \sin a \cdot \sin b$

3. $\tan(a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \cdot \tan b}$

Identidades Trigonométricas para el ángulo doble

1. $\sin(2\alpha) = 2\sin\alpha\cos\alpha$
2. $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$
3. $\tan(2\alpha) = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

Ejemplo Demostrar $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

Demostración

$$\frac{2\tan x}{1+\tan^2 x} = \frac{2\tan}{\sec^2 x}$$

$$= \frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}}$$

$$= \frac{2\sin x\cos^2 x}{\cos x}$$

$$= 2\sin x \cdot \cos x$$

$$= \sin 2x$$

Ejemplo Demostrar $\dfrac{1+\cos 3t}{\sin 3t} + \dfrac{\sin 3t}{1+\cos 3t} = 2\csc 3$

Demostración

$$\frac{1+\cos 3t}{\sin 3t} + \frac{\sin 3t}{1+\cos 3t} = \frac{(1+\cos 3t)^2 + \sin^2 3t}{\sin 3t \cdot (1+\cos 3t)}$$

$$\frac{1 + 2\cos 3t + \cos^2 3t + \sin^2 3t}{\sin 3t \cdot \cos(1+\cos 3t)}$$ recuerde que $\cos^2 3t + \sin^2 3t = 1$

$$= \frac{1 + 2\cos t + 1}{\sin 3t \cdot \cos(1+\cos 3t)}$$

$$= \frac{2(1+\cos 3t)}{\sin 3t \cdot \cos(1+\cos 3t)}$$ factor común 2

$$= \frac{2}{\sin 3t}$$

$$= 2\csc 3t$$

Ejemplo Demostrar $\dfrac{\cot x - \tan x}{\sin x + \cos x} = \csc x - \sec x$

Demostración

$$\frac{\cot x - \tan x}{\sin x + \cos x} = \frac{\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}}{\dfrac{\sin x + \cos x}{1}}$$

$$= \frac{\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}}{\dfrac{\sin x + \cos x}{1}}$$

$$= \frac{(\cos^2 x - \sin^2 x)}{\sin x \cos x(\sin x + \cos x)}$$

$$= \frac{(\cos x + \sin x)(\cos x - \sin x)}{\sin x \cos x(\sin x + \cos x)}$$

$$= \frac{\cos x - \sin x}{\sin x \cos x}$$

$$= \frac{\cos x}{\sin x \cos x} - \frac{\sin x}{\sin x \cos x}$$

$$= \frac{1}{\sin x} - \frac{1}{\cos x}$$

$$= \csc x - \sec x$$

Ejemplo . Demostrar $\dfrac{\sec^2 x}{2 - \sec^2 x} = \sec 2x$

Demostración

$$\frac{\sec^2 x}{2 - \sec^2 x} = \frac{\dfrac{1}{\cos^2 x}}{2 - \dfrac{1}{\cos^2 x}}$$

$$= \frac{\dfrac{1}{\cos^2 x}}{\dfrac{2\cos^2 x - 1}{\cos^2 x}}$$

$$= \frac{\cos^2 x}{\cos^2 x(2\cos^2 x - 1)}$$

$$= \frac{1}{2\cos^2 x - 1}$$

recuerde que $\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha = \cos^2 x - (1 - \cos^2 x) = 2\cos x - 1$

$$\Rightarrow \frac{1}{2\cos^2 x - 1} = \frac{1}{\cos 2x} = \sec 2x$$

Ejemplo. Demostrar $\dfrac{2}{\tan\beta+\cot\beta}=\sin(2\beta)$

Demostración

$$\frac{2}{\tan\beta+\cot\beta}=\frac{2}{\dfrac{\sin\beta}{\cos\beta}+\dfrac{\cos\beta}{\sin\beta}}=\frac{2}{\dfrac{\sin^2\beta+\cos^2\beta}{\cos\beta\sin\beta}}=\frac{2\cos\beta\sin\beta}{\sin^2\beta+\cos^2\beta}=2\cos\beta\sin\beta=\sin 2\beta$$

Ejemplo. Demostrar $\cot(2\alpha)=\dfrac{1}{2}\left(\cot\alpha-\tan\alpha\right)$

Demostración

$$\cot(2\alpha)=\frac{\cos(2\alpha)}{\sin(2\alpha)}=\frac{\cos^2\alpha-\sin^2 x}{2\sin\alpha\cos\alpha}$$

$$=\frac{\cos^2\alpha}{2\sin\alpha\cos\alpha}-\frac{\sin^2\alpha}{2\sin\alpha\cos\alpha}$$

$$=\frac{\cos\alpha}{2\sin\alpha}-\frac{\sin\alpha}{2\cos\alpha}\qquad \text{factor común } \frac{1}{2}$$

$$=\frac{1}{2}(\cot\alpha-\tan\alpha)$$

Ejemplo. Demostrar $\dfrac{\cot x-\cos x}{\cos^3 x}=\dfrac{\csc x}{1+\sin x}$

Demostración

$$\frac{\cot x-\cos x}{\cos^3 x}=\frac{\cot x}{\cos^3 x}-\frac{\cos x}{\cos^3 x}$$

$$=\frac{\dfrac{\cos x}{\sin x}}{\cos^3 x}-\frac{1}{\cos^2 x}$$

$$=\frac{\cos x}{\sin x\cos^3 x}-\frac{1}{\cos^2 x}$$

$$=\frac{1}{\sin x\cos^2 x}-\frac{1}{\cos^2 x}$$

$$=\frac{1-\sin x}{\cos^2 x\sin x}$$

$$=\frac{1-\sin x}{(1-\sin^2 x)\sin x}$$

$$=\frac{1-\sin x}{\sin x(1+\sin x)(1-\sin x)}$$

$$=\frac{1}{\sin x(1+\sin x)}$$

$$= \frac{1}{\sin x} \cdot \frac{1}{1+\sin x}$$

$$= \csc x \cdot \frac{1}{1+\sin x}$$

$$= \frac{\csc x}{1+\sin x}$$

Ejemplo. Demostrar $\dfrac{\tan\alpha - \cos(\alpha-\pi)}{\sin(2\alpha)} = \dfrac{1}{2}\left(\sec^2\alpha + \csc\alpha\right)$

Demostración

Aplicando las fórmulas para $\cos(\alpha-\pi)$ y para $\sin(2\alpha)$ se tiene que

$$\frac{\tan\alpha - \cos(\alpha-\pi)}{\sin(2\alpha)} = \frac{\tan\alpha - (\cos\alpha\cos\pi + \sin\alpha\sin\pi)}{2\sin\alpha\cos\alpha}$$

se sabe que $\sin\pi = 0$ y que $\cos\pi = -1$

$$\Rightarrow \frac{\tan\alpha - (\cos\alpha\cos\pi + \sin\alpha\sin\pi)}{2\sin\alpha\cos\alpha} = \frac{\tan\alpha - (\cos\alpha\cdot -1 + \sin\alpha\cdot 0)}{2\sin\alpha\cos\alpha}$$

$$= \frac{\tan\alpha + \cos\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{\dfrac{\sin\alpha}{\cos\alpha} + \cos\alpha}{2\sin\alpha\cos\alpha}$$

$$= \frac{\dfrac{\sin\alpha + \cos^2\alpha}{\cos\alpha}}{2\sin\alpha\cos\alpha}$$

$$= \frac{\sin\alpha + \cos^2\alpha}{2\sin\alpha\cos^2\alpha}$$

$$= \frac{\sin\alpha}{2\sin\alpha\cos^2\alpha} + \frac{\cos^2\alpha}{2\sin\alpha\cos^2\alpha}$$

$$= \frac{1}{2\cos^2\alpha} + \frac{1}{2\sin\alpha}$$

$$= \frac{1}{2}\left(\sec^2\alpha + \csc\alpha\right)$$