LÍMITES

18. Calcula $\lim\limits_{x\to\infty}\dfrac{\sqrt{2x+1}-\sqrt{2x-1}}{\sqrt{x+1}-\sqrt{x-1}}$

$$\lim_{x\to\infty}\frac{\sqrt{2x+1}-\sqrt{2x-1}}{\sqrt{x+1}-\sqrt{x-1}}=\frac{\infty-\infty}{\infty-\infty}=\text{Indeterminación}=\text{Eliminamos las raíces}=$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{2x+1}-\sqrt{2x-1}\right)\cdot\left(\sqrt{2x+1}+\sqrt{2x-1}\right)\cdot\left(\sqrt{x+1}+\sqrt{x-1}\right)}{\left(\sqrt{x+1}-\sqrt{x-1}\right)\cdot\left(\sqrt{2x+1}+\sqrt{2x-1}\right)\cdot\left(\sqrt{x+1}+\sqrt{x-1}\right)}=$$

$$=\lim_{x\to\infty}\frac{\left[\left(\sqrt{2x+1}\right)^2-\left(\sqrt{2x-1}\right)^2\right]\cdot\left(\sqrt{x+1}+\sqrt{x-1}\right)}{\left[\left(\sqrt{x+1}\right)^2-\left(\sqrt{x-1}\right)^2\right]\cdot\left(\sqrt{2x+1}+\sqrt{2x-1}\right)}=$$

$$=\lim_{x\to\infty}\frac{\left[2x+1-(2x-1)\right]\cdot\left(\sqrt{x+1}+\sqrt{x-1}\right)}{\left[x+1-(x-1)\right]\cdot\left(\sqrt{2x+1}+\sqrt{2x-1}\right)}=\lim_{x\to\infty}\frac{\left[\cancel{2x}+1-\cancel{2x}+1\right]\cdot\left(\sqrt{x+1}+\sqrt{x-1}\right)}{\left[\cancel{x}+1-\cancel{x}+1\right]\cdot\left(\sqrt{2x+1}+\sqrt{2x-1}\right)}=$$

$$=\lim_{x\to\infty}\frac{\cancel{2}\cdot\left(\sqrt{x+1}+\sqrt{x-1}\right)}{\cancel{2}\cdot\left(\sqrt{2x+1}+\sqrt{2x-1}\right)}=\frac{\infty}{\infty}=\text{Indeterminación}=\text{Dividimos n}^{\text{or}}\text{ y d}^{\text{or}}\text{ entre }\sqrt{x}=$$

$$=\lim_{x\to\infty}\frac{\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x}}}{\dfrac{\sqrt{2x+1}+\sqrt{2x-1}}{\sqrt{x}}}=\lim_{x\to\infty}\frac{\sqrt{\dfrac{x+1}{x}}+\sqrt{\dfrac{x-1}{x}}}{\sqrt{\dfrac{2x+1}{x}}+\sqrt{\dfrac{2x-1}{x}}}=\lim_{x\to\infty}\frac{\sqrt{1+\dfrac{1}{x}}+\sqrt{1-\dfrac{1}{x}}}{\sqrt{2+\dfrac{1}{x}}+\sqrt{2-\dfrac{1}{x}}}=$$

$$=\frac{\sqrt{1}+\sqrt{1}}{\sqrt{2}+\sqrt{2}}=\frac{\cancel{2}}{\cancel{2}\sqrt{2}}=\frac{1}{\sqrt{2}}=\text{Racionalizamos}=\frac{\sqrt{2}}{2}$$

19. Calcula $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2}-\sqrt{5x^2+3}}{x+3}$

$$\lim_{x\to\infty}\frac{\sqrt{x^2+2}-\sqrt{5x^2+3}}{x+3}=\frac{\infty-\infty}{\infty}=\text{Indeterminación}=\text{Eliminamos la raíz}=$$

$$=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+2}-\sqrt{5x^2+3}\right)\cdot\left(\sqrt{x^2+2}+\sqrt{5x^2+3}\right)}{(x+3)\cdot\left(\sqrt{x^2+2}+\sqrt{5x^2+3}\right)}=$$

$$=\lim_{x\to\infty}\frac{\left[\left(\sqrt{x^2+2}\right)^2-\left(\sqrt{5x^2+3}\right)^2\right]}{(x+3)\cdot\left(\sqrt{x^2+2}+\sqrt{5x^2+3}\right)}=\lim_{x\to\infty}\frac{\left[x^2+2-\left(5x^2+3\right)\right]}{(x+3)\cdot\left(\sqrt{x^2+2}+\sqrt{5x^2+3}\right)}=$$

$$=\lim_{x\to\infty}\frac{\left[x^2+2-5x^2-3\right]}{(x+3)\cdot\left(\sqrt{x^2+2}+\sqrt{5x^2+3}\right)}=\lim_{x\to\infty}\frac{-4x^2-1}{(x+3)\cdot\left(\sqrt{x^2+2}+\sqrt{5x^2+3}\right)}=$$

$$=\frac{-\infty}{\infty}=\text{Indeterminación}=\text{Dividimos n}^{\text{or}}\text{ y d}^{\text{or}}\text{ entre }x^2=$$

$$=\lim_{x\to\infty}\frac{\dfrac{-4x^2-1}{x^2}}{\dfrac{(x+3)}{x}\cdot\dfrac{\left(\sqrt{x^2+2}+\sqrt{5x^2+3}\right)}{x}}=\lim_{x\to\infty}\frac{-4-\dfrac{1}{x^2}}{\left(1+\dfrac{3}{x}\right)\cdot\left(\dfrac{\sqrt{x^2+2}}{x}+\dfrac{\sqrt{5x^2+3}}{x}\right)}=$$

$$=\lim_{x\to\infty}\frac{-4-\dfrac{1}{x^2}}{\left(1+\dfrac{3}{x}\right)\cdot\left(\sqrt{\dfrac{x^2+2}{x^2}}+\sqrt{\dfrac{5x^2+3}{x^2}}\right)}=\lim_{x\to\infty}\frac{-4-\dfrac{1}{x^2}}{\left(1+\dfrac{3}{x}\right)\cdot\left(\sqrt{1+\dfrac{2}{x^2}}+\sqrt{5+\dfrac{3}{x^2}}\right)}=$$

$$=\frac{-4}{(1)\cdot\left(\sqrt{1}+\sqrt{5}\right)}=\frac{-4}{\left(1+\sqrt{5}\right)}=\text{Racionalizamos}=\frac{-4\cdot\left(1-\sqrt{5}\right)}{\left(1+\sqrt{5}\right)\cdot\left(1-\sqrt{5}\right)}=\frac{\cancel{-4}\cdot\left(1-\sqrt{5}\right)}{\cancel{-4}}=1-\sqrt{5}$$

20. Calcula

$$\lim_{x\to\infty}\left(\frac{3x^3-x}{3x^3-5}\right)^{\frac{x^2+1}{x}} = 1^{\infty} = Indeterminación = \text{Sumo y resto 1 en la base} =$$

$$= \lim_{x\to\infty}\left(1+\frac{3x^3-x}{3x^3-5}-1\right)^{\frac{x^2+1}{x}} = \lim_{x\to\infty}\left(1+\frac{\cancel{3x^3}-x-\cancel{3x^3}+5}{3x^3-5}\right)^{\frac{x^2+1}{x}} =$$

$$= \lim_{x\to\infty}\left(1+\frac{-x+5}{3x^3-5}\right)^{\frac{x^2+1}{x}} = \text{Pongo y quito en el exponente la inversa de } \frac{-x+5}{3x^3-5} =$$

$$= \lim_{x\to\infty}\left(\left(1+\frac{-x+5}{3x^3-5}\right)^{\frac{3x^3-5}{-x+5}}\right)^{\frac{-x+5}{3x^3-5}\cdot\frac{x^2+1}{x}} = \left(\lim_{x\to\infty}\left(1+\frac{-x+5}{3x^3-5}\right)^{\frac{3x^3-5}{-x+5}}\right)^{\lim\limits_{x\to\infty}\left(\frac{-x+5}{3x^3-5}\cdot\frac{x^2+1}{x}\right)} =$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{-x+5}{3x^3-5}\cdot\frac{x^2+1}{x}\right)} = e^{\lim\limits_{x\to\infty}\left(\frac{-x^3+5x^2-x+5}{3x^4-5x}\right)} = e^{\frac{\infty}{\infty}} = e^{\lim\limits_{x\to\infty}\left(\frac{-x^3}{3x^4}\right)} = e^{\lim\limits_{x\to\infty}\left(\frac{-1}{3x}\right)} = e^{\frac{-1}{\infty}} = e^{0} = 1$$