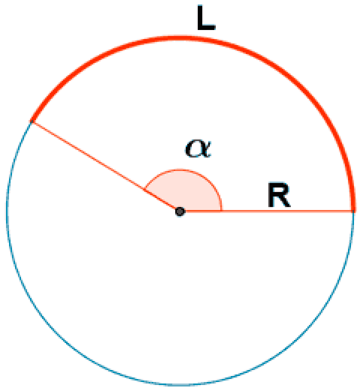
Trigonometría

Medida de ángulos



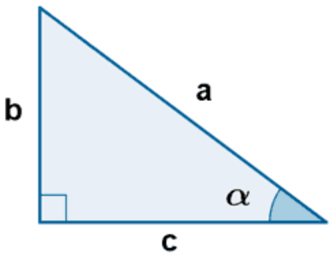
π radianes = 180º

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

L = r α (α en radianes)

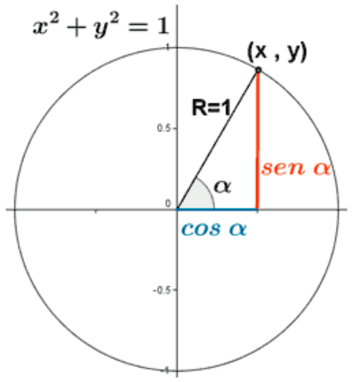
Definición de las funciones trigonométricas



a = hipotenusa b = cateto opuesto c = cateto contiguo

$$\text{sen } \alpha = \frac{b}{a} \qquad \cos \alpha = \frac{c}{a} \qquad \text{tg } \alpha = \frac{\text{sen } \alpha}{\text{sen } \alpha} = \frac{b}{c}$$

$$\text{cosec } \alpha = \frac{1}{\text{sen } \alpha} \qquad \sec \alpha = \frac{1}{\cos \alpha} \qquad \text{cotg } \alpha = \frac{1}{\text{tg } \alpha}$$



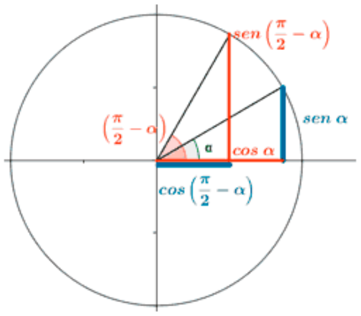
$$\text{sen } \alpha = \frac{y}{R} = y \qquad \cos \alpha = \frac{x}{R} = x \qquad \text{tg } \alpha = \frac{y}{x}$$

$$\text{cosec } \alpha = \frac{1}{\text{sen } \alpha} = \frac{1}{y} \qquad \sec \alpha = \frac{1}{\cos \alpha} = \frac{1}{x} \qquad \text{cotg } \alpha = \frac{1}{\text{tg } \alpha} = \frac{x}{y}$$

Funciones trigonométricas de los ángulos notables

	0 0º	π/6 30º	π/4 45º	π/3 60º	π/2 90º	π 180º	3π/2 270º	2π 360º
seno	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	- 1	0
coseno	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	- 1	0	1
tangente	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	No existe	0	No existe	0

Ángulos complementarios (su suma vale $\pi/2$ radiantes)

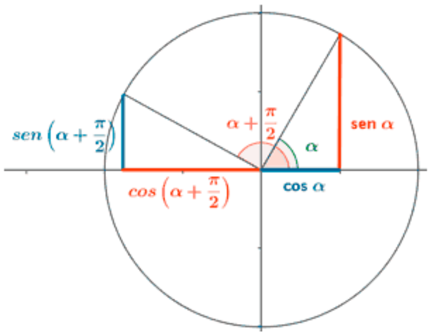


$$\text{sen}\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$$

$$\cos\left(\frac{\pi}{2}-\alpha\right)=\text{sen}\ \alpha$$

$$\text{tg}\left(\frac{\pi}{2}-\alpha\right)=\text{cotg}\ \alpha$$

Ángulos que difieren en $\pi/2$

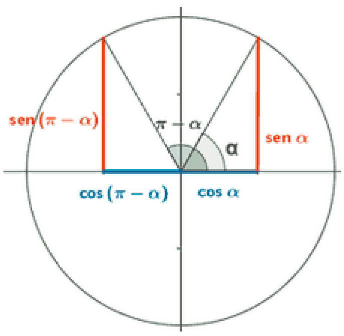


$$\text{sen}\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$$

$$\cos\left(\frac{\pi}{2}+\alpha\right)=-\ \text{sen}\ \alpha$$

$$\text{tg}\left(\frac{\pi}{2}+\alpha\right)=-\ \text{cotg}\ \alpha$$

Ángulos suplementarios (su suma vale π radianes)

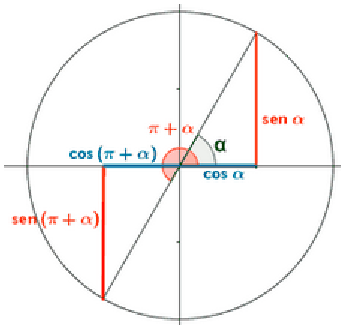


$\text{sen}(\pi - \alpha) = \text{sen}\ \alpha$

$\cos(\pi - \alpha) = -\cos\alpha$

$\text{tg}(\pi - \alpha) = -\ \text{tg}\ \alpha$

Ángulos que se diferencian en π radianes

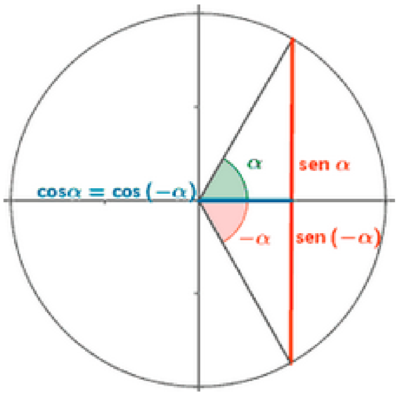


$\text{sen}\ (\pi + \alpha) = -\ \text{sen}\ \alpha$

$\cos\ (\pi + \alpha) = -\cos\alpha$

$\text{tg}\ (\pi + \alpha) = \text{tg}\ \alpha$

Ángulos opuestos



$\text{sen}(-\alpha) = -\text{sen}\ \alpha$

$\cos(-\alpha) = \cos\alpha$

$\text{tg}(-\alpha) = -\text{tg}\ \alpha$

Fórmulas trigonométricas

Ecuación fundamental de la trigonometría

$\text{sen}^2 x + \cos^2 x = 1$ $\quad 1 + \text{tg}^2 x = \sec^2 x$

$1 + \text{cotg}^2 x = \text{cosec}^2 x$

Fórmulas de la suma de ángulos

$\text{sen}(x + y) = \text{sen}\ x \cos y + \cos x\ \text{sen}\ y$

$\cos(x + y) = \cos x \cos y - \text{sen}\ x\ \text{sen}\ y$

$$\text{tg}(x + y) = \frac{\text{tg}\ x + \text{tg}\ y}{1 - \text{tg}\ x\ \text{tg}\ y}$$

Fórmulas de la diferencia de ángulos

$\text{sen}(x - y) = \text{sen}\ x \cos y - \cos x\ \text{sen}\ y$

$\cos(x - y) = \cos x \cos y + \text{sen}\ x\ \text{sen}\ y$

$$\text{tg}(x - y) = \frac{\text{tg}\ x - \text{tg}\ y}{1 + \text{tg}\ x\ \text{tg}\ y}$$

Fórmulas del ángulo doble

$\text{sen}\ 2x = 2\ \text{sen}\ x \cos x$

$\cos 2x = \cos^2 x - \text{sen}^2 x$

$$\text{tg}\ 2x = \frac{2\ \text{tg}\ x}{1 - \text{tg}^2 x}$$

Fórmulas del ángulo mitad

$$\text{sen}\ \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\text{tg}\ \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

El signo ± depende del cuadrante en el que esté.

Fórmulas de reducción de potencias

$$\text{sen}^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\text{tg}^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Sumas y Restas de Senos y Cosenos:

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$$