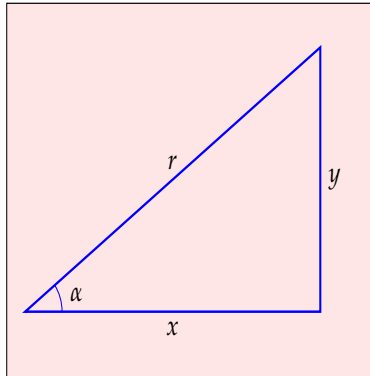


Formulario de Trigonometría



Definiciones Básicas

Funciones principales

$$\begin{aligned} \checkmark \sin \alpha &= \frac{y}{r} \\ \checkmark \cos \alpha &= \frac{x}{r} \\ \checkmark \tan \alpha &= \frac{y}{x} \end{aligned}$$

Funciones recíprocas

$$\begin{aligned} \checkmark \csc \alpha &= \frac{r}{y} \\ \checkmark \sec \alpha &= \frac{r}{x} \\ \checkmark \cot \alpha &= \frac{x}{y} \end{aligned}$$

Identidades recíprocas.

$$\begin{aligned} 1) \sin \alpha &= \frac{1}{\csc \alpha} & 4) \tan \alpha &= \frac{\sin \alpha}{\cos \alpha} \\ 2) \cos \alpha &= \frac{1}{\sec \alpha} & 5) \cot \alpha &= \frac{\cos \alpha}{\sin \alpha} \\ 3) \tan \alpha &= \frac{1}{\cot \alpha} \end{aligned}$$

Propiedades de las funciones trigonométricas.

$$\begin{aligned} 1) \sin \alpha &= \cos(90^\circ - \alpha) & 4) \cot \alpha &= \tan(90^\circ - \alpha) \\ 2) \cos \alpha &= \sin(90^\circ - \alpha) & 5) \csc \alpha &= \sec(90^\circ - \alpha) \\ 3) \tan \alpha &= \cot(90^\circ - \alpha) & 6) \sec \alpha &= \csc(90^\circ - \alpha) \end{aligned}$$

Identidades trigonométricas pitagóricas.

$$\begin{aligned} 1) \sin^2 \alpha + \cos^2 \alpha &= 1 \\ 2) \sec^2 \alpha &= 1 + \tan^2 \alpha \\ 3) \csc^2 \alpha &= 1 + \cot^2 \alpha \end{aligned}$$

Identidades de suma y diferencia de ángulos.

$$\begin{aligned} 1) \sin(\alpha + \beta) &= \sin \alpha \cos \beta + \sin \beta \cos \alpha \\ 2) \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ 3) \sin(\alpha - \beta) &= \sin \alpha \cos \beta - \sin \beta \cos \alpha \\ 4) \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \beta \sin \alpha \\ 5) \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ 6) \tan(\alpha - \beta) &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ 7) \cot(\alpha + \beta) &= \frac{\cot \alpha \cot \beta - 1}{\cot \alpha + \cot \beta} \end{aligned}$$

$$8) \cot(\alpha - \beta) = \frac{\cot \alpha \cot \beta + 1}{\cot \alpha - \cot \beta}$$

Suma de funciones trigonométricas.

$$\begin{aligned} 1) \sin \alpha + \sin \beta &= 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) \\ 2) \sin \alpha - \sin \beta &= 2 \sin \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right) \\ 3) \cos \alpha + \cos \beta &= 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) \\ 4) \cos \alpha - \cos \beta &= 2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right) \end{aligned}$$

Leyes de senos y de cosenos.

$$\begin{aligned} 1) \frac{a}{\sin \alpha} &= \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \\ 2) a^2 &= b^2 + c^2 - 2bc \cos \alpha \\ 3) b^2 &= a^2 + c^2 - 2ac \cos \beta \\ 4) c^2 &= a^2 + b^2 - 2ab \cos \gamma \end{aligned} \quad 5) A = \begin{cases} \frac{ab \sin \gamma}{2} \\ \frac{ac \sin \beta}{2} \\ \frac{bc \sin \alpha}{2} \end{cases}$$

Donde: A es el área del triángulo con lados a, b, c .

Otras Identidades trigonométricas.

$$\begin{aligned} 1) \sin(2\alpha) &= 2 \sin \alpha \cos \alpha & 6) \tan \left(\frac{\alpha}{2} \right) &= \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \\ 2) \cos(2\alpha) &= \cos^2 \alpha - \sin^2 \alpha & 7) \tan \alpha &= \frac{2 \tan \left(\frac{\alpha}{2} \right)}{1 - \tan^2 \left(\frac{\alpha}{2} \right)} \\ 3) \tan(2\alpha) &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} & 8) \sin \alpha &= 2 \sin \left(\frac{\alpha}{2} \right) \cos \left(\frac{\alpha}{2} \right) \\ 4) \cos \left(\frac{\alpha}{2} \right) &= \sqrt{\frac{1 + \cos \alpha}{2}} & 9) \cos \alpha &= \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2} \\ 5) \sin \left(\frac{\alpha}{2} \right) &= \sqrt{\frac{1 - \cos \alpha}{2}} \end{aligned}$$

Resuelve las siguientes igualdades trigonométricas:

1) $\operatorname{sen}^2 x + \operatorname{cos}^2 x = \operatorname{sen} x \operatorname{csc} x$

2) $\frac{1}{\operatorname{csc}^2 x} + \operatorname{cos}^2 x = 1$

3) $\tan^2 x + \operatorname{sen} x \operatorname{csc} x = \sec^2 x$

4) $\frac{\operatorname{cos}^2 x}{\operatorname{sen}^2 x} + 1 = \operatorname{csc}^2 x$

5) $\operatorname{sen}^2 x + \operatorname{cos}^2 x = \operatorname{cos} x \operatorname{sec} x$

6) $\tan^2 x + \tan x \cot x = \sec^2 x$

7) $\frac{\operatorname{sen}^2 x}{\operatorname{cos}^2 x} + 1 = \sec^2 x$

8) $\operatorname{sen}^2 x + \frac{1}{\sec^2 x} = 1$

9) $\tan^2 x \operatorname{cos} x + \operatorname{cos}^2 x = 1$

10) $\operatorname{sen}^2 x + \frac{\operatorname{sen}^2 x}{\tan^2 x} = 1$

11) $\frac{1}{\operatorname{cos} x \operatorname{csc} x} = \tan x$

12) $\operatorname{cos} x \operatorname{csc} x = \cot x$

13) $\frac{1}{\operatorname{sen} x \operatorname{sec} x} = \cot x$

14) $\frac{1}{\tan^2 x} + 1 = \operatorname{csc}^2 x$

15) $\cot^2 x + \frac{1}{\tan x \cot x} = \operatorname{csc}^2 x$

16) $\cot^2 x + \frac{1}{\operatorname{cos} x \operatorname{sec} x} = \operatorname{csc}^2 x$

17) $\tan^2 x + \frac{1}{\operatorname{sen} x \operatorname{csc} x} = \sec^2 x$

18) $\cot^2 x + \operatorname{sen}^2 x + \operatorname{cos}^2 x = \operatorname{csc}^2 x$

Resuelve las siguientes igualdades trigonométricas:

1) $\frac{\operatorname{sen} x}{\operatorname{csc} x} + \frac{\operatorname{cos} x}{\operatorname{sec} x} = 1$

2) $\frac{\operatorname{sec} x}{\tan x + \cot x} = \operatorname{sen} x$

3) $\frac{1 - \operatorname{sen} x}{\operatorname{cos} x} = \frac{\operatorname{cos} x}{1 + \operatorname{sen} x}$

4) $\frac{1 - \operatorname{cos} x}{\operatorname{sen} x} = \frac{\operatorname{sen} x}{1 + \operatorname{cos} x}$

5) $\frac{1}{\operatorname{sec} - \tan x} = \operatorname{sec} x + \tan x$

6) $\frac{1}{\operatorname{csc} x - \cot x} = \operatorname{csc} x + \frac{1}{\tan x}$

7) $\frac{\cot^2 x}{\operatorname{csc} x - 1} = \operatorname{csc} x + \operatorname{sen}^2 x + \operatorname{cos}^2 x$

8) $\frac{\tan x - \operatorname{sen} x}{\operatorname{sen}^3 x} = \frac{\operatorname{sec} x}{1 + \operatorname{cos} x}$

9) $\tan x + \cot x = \frac{1}{\operatorname{sen} x \operatorname{cos} x}$

10) $\operatorname{sec} x + \operatorname{cos}^2 x = \frac{1}{\operatorname{cos} x} + \frac{1}{\sec^2 x}$

11) $\frac{\operatorname{csc} x}{\tan x + \cot x} = \operatorname{cos} x$

12) $(1 - \operatorname{sen}^2 x)(1 + \tan^2 x) = 1$

13) $\frac{1}{1 + \operatorname{sen} x} + \frac{1}{1 - \operatorname{sen} x} = 2 \sec^2 x$

14) $\operatorname{sen} x + \operatorname{cos} x = \operatorname{cos} x(1 + \tan x)$

15) $\cot^2 x + \frac{1}{\operatorname{cos} x \operatorname{sec} x} = \operatorname{csc}^2 x$

16) $\frac{\operatorname{sen}^2 x + \operatorname{cos}^2 x}{\operatorname{sec} x + \tan x} = \operatorname{sec} x - \tan x$

17) $\tan^2 x + \frac{1}{\operatorname{sen} x \operatorname{csc} x} = \sec^2 x$

18) $\cot^2 x + \operatorname{sen}^2 x = \operatorname{csc}^2 x - \operatorname{cos}^2 x$

Resuelve las siguientes ecuaciones trigonométricas:

1. $4 \cos^2 x - 3 = 0$
2. $16 \cos^4 \varphi - 9 = 0$
3. $2 \operatorname{sen}^2 x = -\operatorname{sen} x$
4. $2 \cos^2 x = \operatorname{sen} x - 1$
5. $\frac{\tan x}{\cot x} = 1$
6. $2 \operatorname{sen} \theta + \cos^2 \theta = \frac{7}{4}$
7. $2 \operatorname{sen} x - 1 = 0$
8. $\operatorname{sen} x \cdot \cos x = 0$
9. $(\tan x - 1)(4 \operatorname{sen}^2 x - 3) = 0$
10. $3 \cos^2 x = \operatorname{sen}^2 x$
11. $2 \operatorname{sen} x - \operatorname{csc} x = 1$
12. $2 \sec \theta = \tan \theta + \cot \theta$
13. $\sec^2 x - 4 = 0$
14. $\sqrt{3} + 2 \operatorname{sen} \theta = 0$
15. $\cot^2 x - 3 = 0$
16. $(2 \operatorname{sen} \phi + 1)(2 \cos \phi + 3) = 0$
17. $2 - 8 \cos^2 \mu = 0$
18. $2 \operatorname{sen}^2 \theta = 1 - \operatorname{sen} \theta$
19. $\tan^2 x \cdot \operatorname{sen} x = \operatorname{sen} x$
20. $2 \cos^2 \phi + \cos \phi = 0$
21. $\operatorname{sen}^2 \theta + \operatorname{sen} \theta - 6 = 0$
22. $2 \cos^3 \theta - \cos \theta = 0$
23. $2 \cos \phi + \tan \phi = \sec \phi$
24. $\operatorname{sen} \psi = \tan \psi$
25. $\sec^5 \alpha = 4 \sec \alpha$
26. $\cos \theta \cot^2 \theta = \cos \theta$
27. $3 \operatorname{sen} \beta = 2 \cos^2 \beta$
28. $\operatorname{sen} \phi + \cos^2 \phi = \frac{1}{4}$
29. $\operatorname{sen}^4 \phi - \cos^4 \phi = \frac{1}{2}$
30. $4 \cos \psi \operatorname{sen} \psi + 2 \operatorname{sen} \psi - 2 \cos \psi - 1 = 0$
31. $4 \operatorname{sen}^2 \alpha + 8 \operatorname{sen} \alpha + 3 = 0$
32. $\cos^2 \epsilon - \operatorname{sen}^2 \epsilon + \cos \epsilon + 1 = 0$
33. $\operatorname{sen} \delta = \sqrt{3} \cos \delta - 1$
34. $3 \operatorname{sen} \rho \cos \rho - \operatorname{sen} \rho = 0$
35. $\operatorname{sen}^2 \gamma - \cos^2 \gamma = \frac{1}{2}$
36. $\operatorname{sen}^2 \chi - \cos \chi = \frac{1}{4}$
37. $\operatorname{csc} \omega = 1 + \cot^2 \omega$
38. $2 \cos \mu = \tan \mu + \sec \mu$
39. $2 \cos^2 \chi \operatorname{sen}^2 \chi - \cos \chi \operatorname{sen} \chi = 0$
40. $\cos^2 \chi - 3 \operatorname{sen} \chi + 3 \operatorname{sen}^2 \chi = 0$
41. $1 - \operatorname{sen} \theta = \sqrt{3} \cos \theta$
42. $\cos \alpha + \operatorname{sen} \alpha = 1$
43. $\cos \theta - \operatorname{sen} \theta = 1$
44. $2 \tan \beta - \sec^2 \beta = 0$
45. $\tan \chi + \sec \chi = 1$
46. $\sec \chi + \tan \chi = 0$
47. $2 \operatorname{sen}^3 \chi + \operatorname{sen}^2 \chi - 2 \operatorname{sen} \chi - 1 = 0$
48. $2 \tan \kappa \operatorname{csc} \kappa + 2 \operatorname{csc} \kappa + \tan \kappa + 1 = 0$
49. $2 \operatorname{sen} \chi \operatorname{csc} \chi - \operatorname{csc} \chi = 4 \operatorname{sen} \chi - 2$
50. $\operatorname{sen} \alpha - \cos \alpha = 0$
51. $12 \operatorname{sen}^2 \mu - 5 \operatorname{sen} \mu - 2 = 0$
52. $\operatorname{csc} \alpha + \cot \alpha = \sqrt{3}$
53. $\cos \kappa - \sqrt{3} \operatorname{sen} \kappa = 1$
54. $2 \cos \chi = 1 - \operatorname{sen} \chi$
55. $2 \cos x + 3 \operatorname{sen} x = 2$
56. $3 \operatorname{sen} x + 5 \cos x + 5 = 0$
57. $1 + \operatorname{sen} x = 2 \cos x$
58. $3 \operatorname{sen} x + 4 \cos x = 2$
59. $\operatorname{sen} 3x = \frac{-\sqrt{2}}{2}$
60. $\cos \frac{x}{2} = \frac{1}{2}$
61. $\operatorname{sen} 2x = \frac{-\sqrt{3}}{2}$
62. $\tan 3x = 1$
63. $\cos \frac{x}{2} = \frac{\sqrt{3}}{2}$
64. $\operatorname{sen} 2x = \cos 2x$
65. $\operatorname{sen} 2x = \cos 4x$
66. $\operatorname{sen} 3x = \cos 2x$
67. $\tan 4x = \cot 6x$