

Bloque III. Sistema de ecuaciones

Tema 3 Método de eliminación de Gauss-Jordan

Ejercicios resueltos

III.3-1 Resolver los siguientes sistemas, utilizando el método de eliminación de Gauss:

$$a) \left. \begin{array}{l} 2x + 6y + z = 7 \\ x + 2y - z = -1 \\ 5x + 7y - 4z = 9 \end{array} \right\}$$

$$b) \left. \begin{array}{l} 2x - 3y = 8 \\ 4x - 5y + z = 15 \\ 2x + 4z = 1 \end{array} \right\}$$

$$c) \left. \begin{array}{l} x + y - z = 2 \\ 3x + 3y + z = 2 \\ x + z = 0 \end{array} \right\}$$

$$d) \left. \begin{array}{l} 2x - y = 0 \\ -x + 2y - z = 0 \\ -y + 2z - t = 0 \\ -z + 2t = 5 \end{array} \right\}$$

$$e) \left. \begin{array}{l} x + y + z = 0 \\ x + 2y + 3z = 0 \\ 3x + 5y + 7z = 1 \end{array} \right\}$$

Solución

$$a) \left. \begin{array}{l} 2x + 6y + z = 7 \\ x + 2y - z = -1 \\ 5x + 7y - 4z = 9 \end{array} \right\} \Rightarrow \begin{pmatrix} 2 & 6 & 1 \\ 1 & 2 & -1 \\ 5 & 7 & -4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 9 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & 6 & 1 & 7 \\ 1 & 2 & -1 & -1 \\ 5 & 7 & -4 & 9 \end{array} \right) \xrightarrow{F_2 \leftrightarrow F_1} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & 6 & 1 & 7 \\ 5 & 7 & -4 & 9 \end{array} \right) \xrightarrow{\begin{array}{l} F_2 \rightarrow F_2 - 2F_1 \\ F_3 \rightarrow F_3 - 5F_1 \end{array}}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right) \xrightarrow{\substack{F_2 \rightarrow F_2/2 \\ F_3 \rightarrow F_3 + 3(F_2/2)}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & 3/2 & 9/2 \\ 0 & 0 & 11/2 & 55/2 \end{array}\right) \xrightarrow{\substack{F_1 \rightarrow F_1 - 2F_2 \\ F_3 \rightarrow \frac{2}{11}F_3}}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & -10 \\ 0 & 1 & 3/2 & 9/2 \\ 0 & 0 & 1 & 5 \end{array}\right) \xrightarrow{\substack{F_1 \rightarrow F_1 + 4F_3 \\ F_2 \rightarrow F_2 - \frac{3}{2}F_3}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right) \Rightarrow \begin{cases} x = 10 \\ y = -3 \\ z = 5 \end{cases}$$

b)

$$\left. \begin{array}{rcl} 2x - 3y & = & 8 \\ 4x - 5y + z & = & 15 \\ 2x & + & 4z = 1 \end{array} \right\} \Rightarrow \begin{pmatrix} 2 & -3 & 0 \\ 4 & -5 & 1 \\ 2 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 15 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 4 & -5 & 1 & 15 \\ 2 & 0 & 4 & 1 \end{array}\right) \xrightarrow{\substack{F_2 \rightarrow F_2 - 2F_1 \\ F_3 \rightarrow F_3 - F_1}} \left(\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 0 & 1 & 1 & -1 \\ 0 & 3 & 4 & -7 \end{array}\right) \xrightarrow{F_3 \rightarrow F_3 - 3F_2}$$

$$\xrightarrow{F_3 \rightarrow F_3 - 3F_2} \left(\begin{array}{ccc|c} 2 & -3 & 0 & 8 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -4 \end{array}\right) \Rightarrow \begin{cases} x = \frac{17}{2} \\ y = 3 \\ z = -4 \end{cases}$$

c)

$$\left. \begin{array}{rcl} x + y - z & = & 2 \\ 3x + 3y + z & = & 2 \\ x & + & z = 0 \end{array} \right\} \Rightarrow \begin{pmatrix} 1 & 1 & -1 \\ 3 & 3 & 1 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 3 & 3 & 1 & 2 \\ 1 & 0 & 1 & 0 \end{array}\right) \xrightarrow{\substack{F_2 \rightarrow F_2 - 3F_1 \\ F_3 \rightarrow F_3 - F_1}} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 0 & 4 & -4 \\ 0 & -1 & 2 & -2 \end{array}\right) \xrightarrow{F_2 \leftrightarrow F_3}$$

$$\xrightarrow{F_2 \leftrightarrow F_3} \left(\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & 4 & -4 \end{array}\right) \Rightarrow \begin{cases} x = 1 \\ y = 0 \\ z = -1 \end{cases}$$

d)

$$\left. \begin{array}{rcl} 2x & - & y = 0 \\ -x & + & 2y - z = 0 \\ -y & + & 2z - t = 0 \\ & -z & + 2t = 5 \end{array} \right\} \Rightarrow \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 0 & 0 & | & 0 \\ -1 & 2 & -1 & 0 & | & 0 \\ 0 & -1 & 2 & -1 & | & 0 \\ 0 & 0 & -1 & 2 & | & 5 \end{pmatrix} \xrightarrow{F_1 \leftrightarrow F_2} \begin{pmatrix} -1 & 2 & -1 & 0 & | & 0 \\ 2 & -1 & 0 & 0 & | & 0 \\ 0 & -1 & 2 & -1 & | & 0 \\ 0 & 0 & -1 & 2 & | & 5 \end{pmatrix} \xrightarrow{F_2 \rightarrow F_2 + 2F_1}$$

$$\begin{pmatrix} -1 & 2 & -1 & 0 & | & 0 \\ 0 & 3 & -2 & 0 & | & 0 \\ 0 & -1 & 2 & -1 & | & 0 \\ 0 & 0 & -1 & 2 & | & 5 \end{pmatrix} \xrightarrow{F_2 \leftrightarrow F_3} \begin{pmatrix} -1 & 2 & -1 & 0 & | & 0 \\ 0 & -1 & 2 & -1 & | & 0 \\ 0 & 3 & -2 & 0 & | & 0 \\ 0 & 0 & -1 & 2 & | & 5 \end{pmatrix} \xrightarrow{F_3 \rightarrow F_3 + 3F_2}$$

$$\begin{pmatrix} -1 & 2 & -1 & 0 & | & 0 \\ 0 & -1 & 2 & -1 & | & 0 \\ 0 & 0 & 4 & -3 & | & 0 \\ 0 & 0 & -1 & 2 & | & 5 \end{pmatrix} \xrightarrow{F_3 \leftrightarrow F_4} \begin{pmatrix} -1 & 2 & -1 & 0 & | & 0 \\ 0 & -1 & 2 & -1 & | & 0 \\ 0 & 0 & -1 & 2 & | & 5 \\ 0 & 0 & 4 & -3 & | & 0 \end{pmatrix} \xrightarrow{F_4 \rightarrow F_4 + 4F_3}$$

$$\begin{pmatrix} -1 & 2 & -1 & 0 & | & 0 \\ 0 & -1 & 2 & -1 & | & 0 \\ 0 & 0 & -1 & 2 & | & 5 \\ 0 & 0 & 0 & 5 & | & 20 \end{pmatrix} \Rightarrow \begin{cases} t = 4 \\ z = 3 \\ y = 2 \\ x = 1 \end{cases}$$

e)

$$\left. \begin{array}{rcl} x & + & y + z = 0 \\ x & + & 2y + 3z = 0 \\ 3x & + & 5y + 7z = 1 \end{array} \right\} \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 1 & 2 & 3 & | & 0 \\ 3 & 5 & 7 & | & 1 \end{pmatrix} \xrightarrow{\begin{matrix} F_2 \rightarrow F_2 - F_1 \\ F_3 \rightarrow F_3 - 3F_1 \end{matrix}} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & 2 & 4 & | & 1 \end{pmatrix} \xrightarrow{F_3 \rightarrow F_3 - 2F_2} \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & 2 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{pmatrix}$$

\Rightarrow **NO EXISTE SOLUCION**

III.3-2 Estudiar y resolver cuando sea posible, los siguientes sistemas:

$$a) \begin{cases} x + y - z = 2 \\ 3x + 3y + z = 2 \\ x + z = 0 \end{cases}$$

$$b) \begin{cases} x + 2y - 5z - t + 2u = -3 \\ y - 2z + t - 4u = 1 \\ 2x - 3y + 4z + 2t - u = 9 \end{cases}$$

$$c) \begin{cases} x + 2y - 3z = 0 \\ -2x - z = -3 \\ -x + y = 0 \\ -2y + 4z = 4 \end{cases}$$

$$d) \begin{cases} x + y - 3t - u = -3 \\ x - y + 2z - t = -1 \\ 4x - 2y + 6z + 3t - 4u = 3 \\ 2x + 4y - 2z + 4t - 7u = 4 \end{cases}$$

Solución

$$a) \begin{cases} x + y - z = 2 \\ 3x + 3y + z = 2 \\ x + z = 0 \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & -1 \\ 3 & 3 & 1 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 & | & 2 \\ 3 & 3 & 1 & | & 2 \\ 1 & 0 & 1 & | & 0 \end{pmatrix} \xrightarrow{\substack{F_2 \rightarrow F_2 - 3F_1 \\ F_3 \rightarrow F_3 - F_1}} \begin{pmatrix} 1 & 1 & -1 & | & 2 \\ 0 & 0 & 4 & | & -4 \\ 0 & -1 & 2 & | & -2 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \text{rango}(A) = 3 \\ \text{rango}(A|B) = 3 \\ \text{variables} = 3 \end{cases} \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 3 \Rightarrow \begin{cases} \text{Sistema} \\ \text{compatible} \\ \text{determinado} \end{cases}$$

$$\begin{cases} x + y - z = 2 \\ z = -1 \\ -y + 2z = -2 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 0 \\ z = -1 \end{cases}$$

$$b) \begin{cases} x + 2y - 5z - t + 2u = -3 \\ y - 2z + t - 4u = 1 \\ 2x - 3y + 4z + 2t - u = 9 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & -5 & -1 & 2 \\ 0 & 1 & -2 & 1 & -4 \\ 2 & -3 & 4 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ t \\ u \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -5 & -1 & 2 & | & -3 \\ 0 & 1 & -2 & 1 & -4 & | & 1 \\ 2 & -3 & 4 & 2 & -1 & | & 9 \end{pmatrix} \xrightarrow{F_3 \rightarrow F_3 - 2F_1} \begin{pmatrix} 1 & 2 & -5 & -1 & 2 & | & -3 \\ 0 & 1 & -2 & 1 & -4 & | & 1 \\ 0 & -7 & 14 & 4 & -5 & | & 15 \end{pmatrix}$$

$$\xrightarrow{F_3 \rightarrow F_3 + 7F_2} \begin{pmatrix} 1 & 2 & -5 & -1 & 2 & | & -3 \\ 0 & 1 & -2 & 1 & -4 & | & 1 \\ 0 & 0 & 0 & 11 & -33 & | & 22 \end{pmatrix} \Rightarrow$$

$$\begin{cases} \text{rango}(A) = 3 \\ \text{rango}(A|B) = 3 \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 3 < 5 \\ \text{variables} = 5 \end{cases} \Rightarrow \begin{cases} \text{Sistema} \\ \text{compatible} \\ \text{indeterminado} \end{cases}$$

$$\begin{cases} x + 2y - 5z - t + 2u = -3 \\ y - 2z + t - 4u = 1 \\ t - 3u = 2 \end{cases} \Rightarrow \begin{cases} t = 2 + 3u \\ y = 2z + u - 1 \\ x = z - u + 1 \end{cases}$$

c)

$$\begin{cases} x + 2y - 3z = 0 \\ -2x - z = -3 \\ -x + y = 0 \\ -2y + 4z = 4 \end{cases} \Rightarrow \begin{pmatrix} 1 & 2 & -3 \\ -2 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 0 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -3 & | & 0 \\ -2 & 0 & -1 & | & -3 \\ -1 & 1 & 0 & | & 0 \\ 0 & -2 & 4 & | & 4 \end{pmatrix} \xrightarrow{\begin{matrix} F_2 \rightarrow F_2 + 2F_1 \\ F_3 \rightarrow F_3 + F_1 \end{matrix}} \begin{pmatrix} 1 & 2 & -3 & | & 0 \\ 0 & 4 & -7 & | & -3 \\ 0 & 3 & -3 & | & 0 \\ 0 & -2 & 4 & | & 4 \end{pmatrix} \xrightarrow{F_4 \leftrightarrow F_2}$$

$$\begin{pmatrix} 1 & 2 & -3 & | & 0 \\ 0 & -2 & 4 & | & 4 \\ 0 & 3 & -3 & | & 0 \\ 0 & 4 & -7 & | & -3 \end{pmatrix} \xrightarrow{F_2 \rightarrow -\frac{1}{2}F_2} \begin{pmatrix} 1 & 2 & -3 & | & 0 \\ 0 & 1 & -2 & | & -2 \\ 0 & 3 & -3 & | & 0 \\ 0 & 4 & -7 & | & -3 \end{pmatrix} \xrightarrow{\begin{matrix} F_3 \rightarrow F_3 - 3F_2 \\ F_4 \rightarrow F_4 - 4F_2 \end{matrix}}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -3 & 0 & 0 \\ 0 & 1 & -2 & -2 & -2 \\ 0 & 0 & 3 & 6 & 6 \\ 0 & 0 & 1 & 5 & 5 \end{array} \right) \xrightarrow{F_3 \leftrightarrow F_4} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 0 & 0 \\ 0 & 1 & -2 & -2 & -2 \\ 0 & 0 & 1 & 5 & 5 \\ 0 & 0 & 3 & 6 & 6 \end{array} \right) \xrightarrow{F_4 \rightarrow F_4 - 3F_3} \left(\begin{array}{cccc|c} 1 & 2 & -3 & 0 & 0 \\ 0 & 1 & -2 & -2 & -2 \\ 0 & 0 & 1 & 5 & 5 \\ 0 & 0 & 0 & -9 & -9 \end{array} \right)$$

$$\Rightarrow \begin{cases} \text{rango}(A) = 3 \\ \text{rango}(A|B) = 4 \Rightarrow \text{rango}(A) \neq \text{rango}(A|B) \\ \text{variables} = 3 \end{cases} \Rightarrow \begin{cases} \text{Sistema} \\ \text{incompatible} \end{cases}$$

$$\text{d) } \begin{cases} x + y - 3t - u = -3 \\ x - y + 2z - t = -1 \\ 4x - 2y + 6z + 3t - 4u = 3 \\ 2x + 4y - 2z + 4t - 7u = 4 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -1 & -3 \\ 1 & -1 & 2 & -1 & 0 & -1 \\ 4 & -2 & 6 & 3 & -4 & 3 \\ 2 & 4 & -2 & 4 & -7 & 4 \end{array} \right) \xrightarrow{\begin{matrix} F_2 \rightarrow F_2 - F_1 \\ F_3 \rightarrow F_3 - 4F_1 \\ F_4 \rightarrow F_4 - 2F_1 \end{matrix}} \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -1 & -3 \\ 0 & -2 & 2 & 2 & 1 & 2 \\ 0 & -6 & 6 & 15 & 0 & 15 \\ 0 & 2 & -2 & 10 & -5 & 10 \end{array} \right)$$

$$\xrightarrow{\begin{matrix} F_3 \rightarrow F_3 - 3F_2 \\ F_4 \rightarrow F_4 + F_2 \end{matrix}} \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -1 & -3 \\ 0 & -2 & 2 & 2 & 1 & 2 \\ 0 & 0 & 0 & 9 & -3 & 9 \\ 0 & 0 & 0 & 12 & -4 & 12 \end{array} \right) \xrightarrow{\begin{matrix} F_3 \rightarrow \frac{1}{3}F_3 \\ F_4 \rightarrow \frac{1}{4}F_4 \end{matrix}} \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -1 & -3 \\ 0 & -2 & 2 & 2 & 1 & 2 \\ 0 & 0 & 0 & 3 & -1 & 3 \\ 0 & 0 & 0 & 3 & -1 & 3 \end{array} \right)$$

$$\xrightarrow{F_4 \rightarrow F_4 - F_3} \left(\begin{array}{ccccc|c} 1 & 1 & 0 & -3 & -1 & -3 \\ 0 & -2 & 2 & 2 & 1 & 2 \\ 0 & 0 & 0 & 3 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow$$

$$\begin{cases} \text{rango}(A) = 3 \\ \text{rango}(A|B) = 3 \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 3 < 5 \\ \text{variables} = 5 \end{cases} \Rightarrow \begin{cases} \text{Sistema} \\ \text{compatible} \\ \text{indeterminado} \end{cases}$$

$$\begin{cases} x + y - 3t - u = -3 \\ -2y + 2z + 2t + u = 2 \\ 3t - u = 3 \end{cases} \Rightarrow \begin{cases} x = \frac{7t - 2z - 7}{2} \\ y = \frac{2z + 5t - 5}{2} \\ u = 3t - 3 \end{cases}$$

III.3-3 Estudiar y resolver, cuando sea posible, los siguientes sistemas en función del valor de los parámetros:

$$a) \begin{cases} x + y + 2z = 2 \\ 2x + 3y - z = 5 \\ 3x + 4y + z = c \end{cases}$$

$$b) \begin{cases} x - y + 1 = 0 \\ x - z + 2 = 0 \\ x + y + az = -b \end{cases}$$

$$c) \begin{cases} ax + y + z = 1 \\ x + ay + z = a \\ x + y + az = a^2 \end{cases}$$

Solución

a)

$$\begin{cases} x + y + 2z = 2 \\ 2x + 3y - z = 5 \\ 3x + 4y + z = c \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & -1 \\ 3 & 4 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ c \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 2 & | & 2 \\ 2 & 3 & -1 & | & 5 \\ 3 & 4 & 1 & | & c \end{pmatrix} \xrightarrow[\substack{F_2 \rightarrow F_2 - 2F_1 \\ F_3 \rightarrow F_3 - 3F_1}]{\rightarrow} \begin{pmatrix} 1 & 1 & 2 & | & 2 \\ 0 & 1 & -5 & | & 1 \\ 0 & 1 & -5 & | & c-6 \end{pmatrix} \xrightarrow{F_3 \rightarrow F_3 - F_2} \begin{pmatrix} 1 & 1 & 2 & | & 2 \\ 0 & 1 & -5 & | & 1 \\ 0 & 0 & 0 & | & c-7 \end{pmatrix}$$

$$i) \text{ Si } c \neq 7 \Rightarrow \begin{cases} \text{rango}(A) = 2 \\ \text{rango}(A|B) = 3 \Rightarrow \text{rango}(A) \neq \text{rango}(A|B) \\ \text{variables} = 3 \end{cases}$$

$$\Rightarrow \begin{cases} \text{Sistema} \\ \text{incompatible} \end{cases}$$

$$ii) \text{ Si } c = 7 \Rightarrow \begin{cases} \text{rango}(A) = 2 \\ \text{rango}(A|B) = 2 \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 2 < 3 \\ \text{variables} = 3 \end{cases}$$

$$\Rightarrow \begin{cases} \text{Sistema} \\ \text{compatible} \\ \text{indeterminado} \end{cases} \Rightarrow \begin{cases} x + y + 2z = 2 \\ y - 5z = 1 \end{cases} \Rightarrow \begin{cases} x = 1 - 7z \\ y = 5z + 1 \end{cases}$$

b)

$$\left. \begin{array}{l} x - y + 1 = 0 \\ x - z + 2 = 0 \\ x + y + az = -b \end{array} \right\} \Rightarrow \begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & a \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -b \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & 0 & | & -1 \\ 1 & 0 & -1 & | & -2 \\ 1 & 1 & a & | & -b \end{pmatrix} \xrightarrow{\substack{F_2 \rightarrow F_2 - F_1 \\ F_3 \rightarrow F_3 - F_1}} \begin{pmatrix} 1 & -1 & 0 & | & -1 \\ 0 & 1 & -1 & | & -1 \\ 0 & 2 & a & | & 1-b \end{pmatrix} \xrightarrow{F_3 \rightarrow F_3 - 2F_2}$$

$$\xrightarrow{F_3 \rightarrow F_3 - 2F_2} \begin{pmatrix} 1 & -1 & 0 & | & -1 \\ 0 & 1 & -1 & | & -1 \\ 0 & 0 & a+2 & | & 3-b \end{pmatrix} \Rightarrow$$

$$i) \text{ Si } a \neq -2 \Rightarrow \begin{cases} \text{rango}(A) = 3 \\ \text{rango}(A|B) = 3 \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 3 \\ \text{variables} = 3 \end{cases}$$

$$\Rightarrow \begin{cases} \text{Sistema} & x - y = -1 \\ \text{compatible} & y - z = -1 \\ \text{determinado} & (a+2)z = 3-b \end{cases} \Rightarrow \begin{cases} x = \frac{-1-2a-b}{a+2} \\ y = \frac{1-a-b}{a+2} \\ z = \frac{3-b}{a+2} \end{cases}$$

$$ii) \text{ Si } a = -2 \Rightarrow \text{rango}(A) = 2$$

$$\bullet) \text{ Si } b \neq 3 \Rightarrow \text{rango}(A|B) = 3$$

$$\begin{cases} \text{rango}(A) = 2 \\ \text{rango}(A|B) = 3 \Rightarrow \text{rango}(A) \neq \text{rango}(A|B) \Rightarrow \\ \text{variables} = 3 \end{cases} \Rightarrow \begin{cases} \text{Sistema} \\ \text{incompatible} \end{cases}$$

$$\bullet) \text{ Si } b = 3 \Rightarrow \text{rango}(A|B) = 2$$

$$\begin{cases} \text{rango}(A) = 2 \\ \text{rango}(A|B) = 2 \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 2 < 3 \\ \text{variables} = 3 \end{cases} \Rightarrow \begin{cases} \text{Sistema} \\ \text{compatible} \\ \text{indeterminado} \end{cases} \Rightarrow \begin{cases} x - y = -1 \\ y - z = -1 \end{cases} \Rightarrow \begin{cases} x = z - 2 \\ y = z - 1 \end{cases}$$

$$c) \quad \left. \begin{array}{l} ax + y + z = 1 \\ x + ay + z = a \\ x + y + az = a^2 \end{array} \right\} \Rightarrow \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix}$$

$$\begin{pmatrix} a & 1 & 1 & | & 1 \\ 1 & a & 1 & | & a \\ 1 & 1 & a & | & a^2 \end{pmatrix} \xrightarrow{F_1 \leftrightarrow F_2} \begin{pmatrix} 1 & a & 1 & | & a \\ a & 1 & 1 & | & 1 \\ 1 & 1 & a & | & a^2 \end{pmatrix} \xrightarrow{\begin{array}{l} F_2 \rightarrow F_2 - aF_1 \\ F_3 \rightarrow F_3 - F_1 \end{array}}$$

$$\begin{pmatrix} 1 & a & 1 & | & a \\ 0 & 1-a^2 & 1-a & | & 1-a^2 \\ 0 & 1-a & a-1 & | & a^2-a \end{pmatrix} \xrightarrow{F_2 \leftrightarrow F_3} \begin{pmatrix} 1 & a & 1 & | & a \\ 0 & 1-a & a-1 & | & a^2-a \\ 0 & 1-a^2 & 1-a & | & 1-a^2 \end{pmatrix} \xrightarrow{F_3 \rightarrow F_3 - (1+a)F_2}$$

$$\begin{pmatrix} 1 & a & 1 & | & a \\ 0 & 1-a & a-1 & | & a^2-a \\ 0 & 0 & -a^2-a+2 & | & (1+a)^2(1-a) \end{pmatrix} \Rightarrow \begin{cases} -a^2-a+2=0 \Rightarrow \begin{cases} a=1 \\ a=-2 \end{cases} \\ (1+a)^2(1-a)=0 \Rightarrow \begin{cases} a=1 \\ a=-1 \end{cases} \end{cases}$$

i) Si $a \neq 1, -2 \Rightarrow$

$$\begin{cases} \text{rango}(A) = 3 \\ \text{rango}(A|B) = 3 \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 3 \Rightarrow \\ \text{variables} = 3 \end{cases} \Rightarrow \begin{cases} \text{Sistema} \\ \text{compatible} \\ \text{determinado} \end{cases}$$

ii) Si $a = 1 \Rightarrow \begin{cases} \text{rango}(A) = 1 \\ \text{rango}(A|B) = 1 \Rightarrow \text{rango}(A) = \text{rango}(A|B) = 1 < 3 \\ \text{variables} = 3 \end{cases}$

$$\Rightarrow \begin{cases} \text{Sistema} \\ \text{compatible} \\ \text{indeterminado} \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \Rightarrow x + y + z = 1 \Rightarrow x = 1 - y - z$$

iii) Si $a = -2 \Rightarrow \begin{cases} \text{rango}(A) = 2 \\ \text{rango}(A|B) = 3 \Rightarrow \text{rango}(A) \neq \text{rango}(A|B) \\ \text{variables} = 3 \end{cases}$

$$\Rightarrow \begin{cases} \text{Sistema} \\ \text{incompatible} \end{cases} \Rightarrow \begin{pmatrix} 1 & -2 & 1 & | & -2 \\ 0 & 3 & -3 & | & 6 \\ 0 & 0 & 0 & | & 3 \end{pmatrix}$$